Timing is important: Risk-aware Fund Allocation based on Time-Series Forecasting

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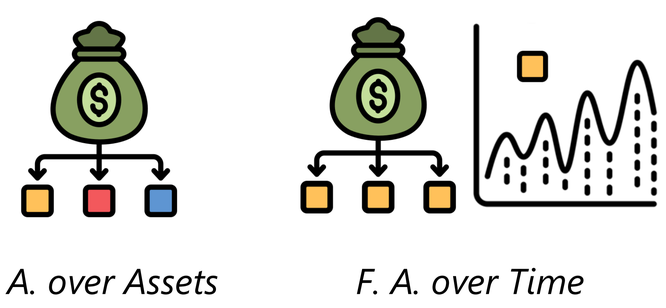
Fund Allocation, but over time



F. A. over Assets *F. A. over Time*

E.g., Buy X USD over one day

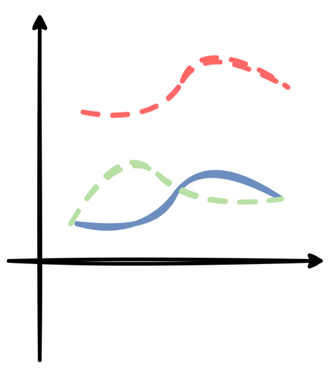
A Predict-then-Optimize (PtO) framework:

- A forecasting model: $y_T = M(x_T)$
- An allocation model: $\min_a a * y_T, \quad \text{s.t.} \sum a = 1$

From PtO to PnO

The Drawbacks of PtO:

- Gap between *Prediction* and *Optimization* Stage



Goal Mismatch:
- Prediction: Green > Red
- Optimization: Red > Green

Pass Information from Optimization to Prediction

$$\ell_o(\mathbf{a}^*(\hat{y}_T)) \triangleq 2\mathbf{a}^*(y_T)\hat{y}_T - \mathbf{a}^*(y_T)y_T + \max_{\mathbf{a}\in\mathcal{A}}\{\mathbf{a}y_T - 2\mathbf{a}\hat{y}_t\}.$$

SPO+ (Surrogate) Loss for alignment

- The Uncertainty of the Forecasting

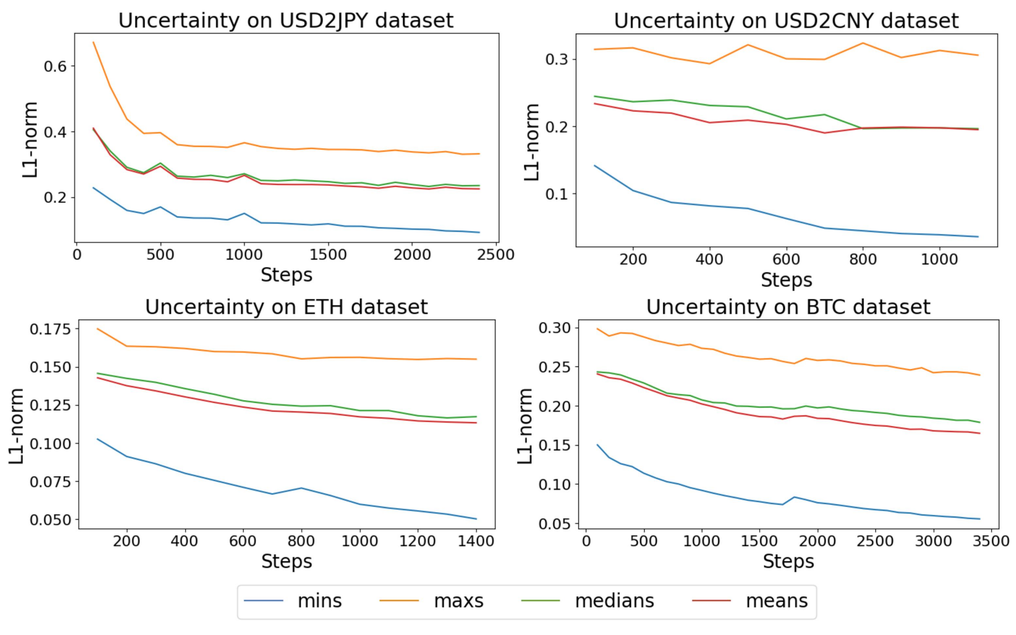
Algorithm 1 Calculating Positional Uncertainty for Forecasting Model

Require: Calibration Dataset $\mathcal{D}_c$, coverage rate γ
Ensure: Positional Uncertainty $\mathbf{r}$

```
Initialize Positional Uncertainty Sets ε_1 = { }, ··· , ε_H = { }
for for data instance (x_T, y_T, c_T) in Calibration Set D_c do
    Calculate ŷ_T = [p̂_{T+1}, ··· p̂_{T+H}] given Eq. 2
    for h in 1, ··· , H do
        ε_h ← ε_h ∪ {|p̂_{T+h} − p_{T+h}|}
for h in 1, ··· , H do
    r_h = ((|D_c|+1)/|D_c| γ) - quantile in ε_h
Return r = [r_1, r_2, ··· , r_H]
```

Positional-aware risk on forecasting

Why not having a fixed threshold



Experiment

Category	Dataset	Forecasting-Only Top-1 regret↓	Forecasting-Only Top-1 R.R.↓	Forecasting-Only Top-5 regret↓	Forecasting-Only Top-5 R.R.↓	Risk-Avoid Top-1 regret↓	Risk-Avoid Top-1 R.R.↓	Risk-Avoid Top-5 regret↓	Risk-Avoid Top-5 R.R.↓	RTS-PtO regret↓	RTS-PtO R.R.↓	RTS-PnO regret↓	RTS-PnO R.R.↓	Relative Improvement regret(%)	Relative Improvement R.R.(%)
Currency	USD2CNY	36.88	5.10	37.00	5.12	35.80	4.95	35.83	4.96	35.74	4.94	**31.68**	**4.38**	12.82%	12.79%
	USD2JPY	54.50	34.92	54.21	34.73	49.66	31.90	50.01	32.12	52.11	32.66	**48.77**	**31.25**	1.82%	2.08%
	AUD2USD	19.56	29.60	19.92	30.15	19.38	29.36	19.49	29.52	19.48	29.51	**19.06**	**28.84**	1.68%	1.80%
	NZD2USD	17.43	28.75	17.66	29.14	16.54	27.29	16.64	27.44	16.82	27.75	**15.68**	**25.85**	5.48%	5.57%
Stock	S&P 500	134.99	4.25	135.47	4.24	**122.50**	**3.84**	124.24	3.90	126.06	3.94	124.05	3.90	-1.27%	-1.56%
	Dow Jones	1090.88	4.16	1075.79	4.09	1022.73	3.91	1032.21	3.93	1022.90	3.92	**997.52**	**3.82**	2.53%	2.36%
Cryptos	BTC	2159.78	4.46	2167.96	4.47	1856.21	3.90	1858.57	3.91	1924.65	3.96	**1843.26**	**3.70**	0.70%	5.41%
	ETH	151.14	5.56	149.61	5.48	131.41	**4.68**	131.42	**4.68**	138.60	4.96	**131.40**	4.73	0.00%	-1.07%
Avg. Rank		5.38	5.5	5.63	5.5	2	1.88	3.38	3.13	3.5	3.5	1.13	1.25		

Forecasting Model	Dataset	Forecasting-Only Top-1 regret↓	Forecasting-Only Top-1 R.R.↓	Forecasting-Only Top-5 regret↓	Forecasting-Only Top-5 R.R.↓	Risk-Avoiding Top-1 regret↓	Risk-Avoiding Top-1 R.R.↓	Risk-Avoiding Top-5 regret↓	Risk-Avoiding Top-5 R.R.↓	RTS-PtO regret↓	RTS-PtO R.R.↓	RTS-PnO regret↓	RTS-PnO R.R.↓	Relative Improvement regret(%)	Relative Improvement R.R.(%)
DLinear	USD2CNY	36.99	5.12	36.73	5.08	35.50	4.91	38.11	5.27	35.31	4.98	**34.88**	**4.81**	1.23%	3.50%
	Dow Jones	1103.11	4.21	1128.71	4.24	**1036.65**	**3.96**	1075.97	4.08	1073.30	4.10	1042.35	3.98	-0.55%	-0.51%
TimesNet	USD2CNY	39.77	5.50	39.46	5.46	36.83	5.09	37.47	5.18	35.99	4.98	**33.73**	**4.66**	6.70%	6.87%
	Dow Jones	1157.76	4.40	1143.82	4.32	1037.71	3.98	1082.45	4.11	1042.67	3.95	**972.51**	**3.74**	6.70%	5.61%
FEDFormer	USD2CNY	36.44	5.04	36.89	5.10	36.28	5.02	36.53	5.05	35.94	4.97	**32.32**	**4.47**	11.23%	11.19%
	Dow Jones	1087.49	4.15	1100.99	4.19	1065.08	4.05	1078.61	4.09	1043.41	3.98	**1010.96**	**3.82**	3.21%	4.19%

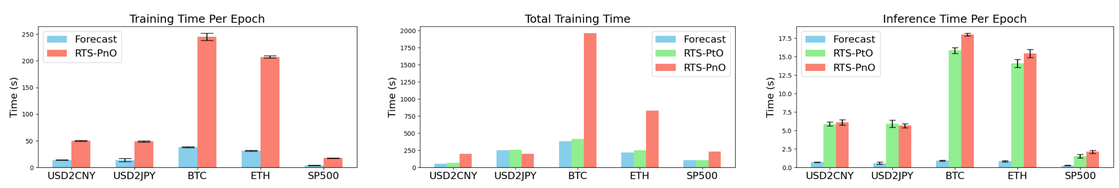
Observations:
- RTS-PnO proves to be effective — *RTS-PnO > others*
- Further Observations on
  a) Forecasting-Only not reliable: — *others > Forecasting-Only*
  b) Risk-Avoid is effective: — *Good on Stock & Crypto*
- RTS-PnO is model-agnostic — *Good on other TS models*

Ablation Study

Dataset	PtO regret↓	PtO R.R.↓	Fixed-PnO regret↓	Fixed-PnO R.R.↓	Adaptive-PnO regret↓	Adaptive-PnO R.R.↓
USD2CNY	35.74	4.94	34.66	4.71	**31.68**	**4.38**
USD2JPY	52.11	32.66	49.21	31.83	**48.77**	**31.25**
S&P 500	126.06	3.94	129.13	4.02	**124.05**	**3.90**
Dow Jones	1022.90	3.92	1026.45	3.92	**997.52**	**3.82**
BTC	1924.65	3.96	1939.81	4.02	**1843.26**	**3.70**
ETH	138.60	4.96	136.66	4.90	**131.40**	**4.73**

Category	Dataset	Prediction MSE	Prediction MAE	RTS-PnO MSE	RTS-PnO MAE
Currency	USD2CNY	**0.0049**	**0.0397**	0.0053	0.0430
	USD2JPY	**0.0383**	**0.1263**	0.1201	0.2796
	AUD2USD	**0.0277**	**0.1220**	0.0350	0.1439
	NZD2USD	**0.0233**	**0.1072**	0.0327	0.1334
Stock	S&P 500	**0.1533**	**0.2744**	0.5567	0.6194
	Dow Jones	**0.1184**	**0.2354**	0.3552	0.4815
Criptos	BTC	**0.0197**	**0.0962**	0.0953	0.2321
	ETH	**0.0213**	**0.1003**	0.1297	0.2608

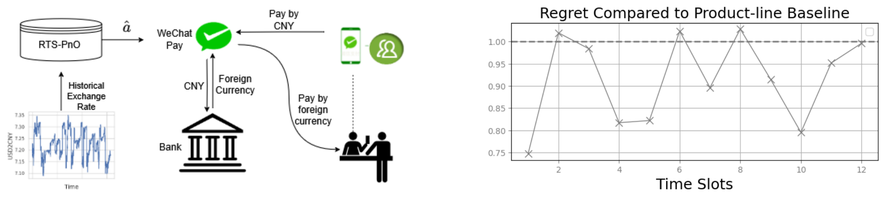
- Ablation on Risk Threshold: *Adaptive > Fixed*
- Ablation on Prediction Performance: *Sacrificed the accuracy*



- Increase on both training and inference time
- Inference time increase on both PtO and PnO
- Training time increase on PnO

Perhaps more efficient optimization can help?

Online Evaluation



- USD cash reverse to provide services during weekend
- An average 8.4% decrease in terms of relative regret

Summary

- Allocation over Time
- RTS-PnO:
  - Risk-aware: time series conformal
  - Predict-then-Optimize
- Evaluated both online and offline

Paper Code